

General Convex Topological Spaces and Fixed Points

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ABSTRACT. In this paper we shall consider general convexity which is described in an abstract from topological spaces. Also, we formulate a fixed point theorem for general nonexpansive mappings in the general convex topological spaces. This result extends previous results of M. Brodskij, D. Milman, W. Kirk, K. Goebel, W. Takashi, F. Browder, D. Göhde and some others.

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